Benha Journal of Applied Sciences (BJAS) print: ISSN 2356–9751
Vol. (6) Issue (6) Part (2) (2021), (309-318) online: ISSN 2356–976x
http://bjas.journals.ekb.eg

A Theoretical investigation of Magnetohydrodynamics flow and the heat transfer process of a fluid between two porous discs

M. Abdel Wahab, Mostafa.Y.El-Bakry, S.E.E.Hamza and A.G.El-ashhab
Physics Dept., Faculty of Science, Benha Univ., Benha, Egypt
E-Mail**:** aya.gamal@fsc.bu.edu.eg

Abstract

In the present study, a theoretical investigation is considered for the steady incompressible flow of an electrically conducting and viscous fluid between two porous discs, one rotating while the other is a stationary disc with a constant uniform suction velocity W_1 on the surface of both discs. An axial magnetic field is subjected to the fluid. The problem of heat transfer process and the temperature distribution for the flow field has been investigated. The formulation of the problem, the basic governing equations in the suitable system of coordinates and the appropriate boundary conditions that govern the fluid motion have been obtained. By using the suitable similarity transformation, the governing nonlinear partial differential equations of motion are transformed into a dimensionless nonlinear ordinary differential equations that solved by using an analytical approximation method. The graphical illustrations due to the effect of a various values of the Hartmann number, the Reynolds numbers and the Prandtl number on the fluid velocity and the temperature distributions have been discussed.

Keywords: MHD flow, Porous disc, Heat transfer, Perturbation method, approximation solution.

1. Introduction

The study of fluid flow between parallel porous discs considerably analyzed in the recent years because of its importance in several of technologies and engineering applications. Some important examples of these applications are in the electric power generating system, hydrodynamical machines, machines of food processing, computer storage devices, cooling of turbine blades, gas turbines and the crystal growth process. For the first time Von Karman [1] suggested a method to transform set of a partial differential equations that govern the flow over an infinite rotating disk to nonlinear form and until now many researchers depended on these transformations to make the equations simpler in mathematical handling. Batchelor [2] that extended Von Karman work for studying the solutions of various ranges of the Reynolds numbers based on the angular velocities of the disks. Elcrat [3] studied a boundary value problem that yields an exact solutions of the Navier-Stokes equations for the flow between two infinite coaxial porous disks and proved the theory of existence, uniqueness, and asymptotic behavior of the solution also considering uniform suction or injection on the disks. Rasmussen [4] considered in this paper the flow between two porous coaxial disks and a numerical solutions have been calculated for this problem. Gaur and Chaudhary [5] investigated a problem of the heat transfer process for laminar flow between two parallel porous disks where the flow is due to suction or injection at both the two disks. Rudraiah and Chandrasekhara [6] studied three dimensional MHD flow and the interaction of injection or suction between a rotating and a stationary disk and asymptotic solutions are obtained. A series solutions are given for the small values of Reynolds numbers that introduced by Stewartson [7]. Wang and Watson [8] examined the flow between rotating disks with a uniform injection on the porous disk. Srivastava and Sharma [9] studied the effect of a transverse magnetic field on the flow between two infinite disks by introducing a solution for the rotational Reynolds number. Stephenson [10] numerically and analytically studied Magnetohydrodynamics flow between rotating coaxial disks for arbitrary Hartmann and Reynolds numbers. Kumar, et al. [11] studied the MHD flow of a conducting fluid between a non-porous rotating and a stationary porous disk where the governing equations of motion are solved by using least change secant update quasi-Newton and modern root finding techniques. A numerical study of Magnetohydrodynamics flow between a rotating and a stationary porous coaxial discs examined by Agarwal and Bhargava [12] with a suction on the stationary lower disc. The unsteady flow between two parallel rotating disks of a fluid with the heat transfer is investigated by Ibrahim [13]. An approximate solution presented by Ersoy [14] for the flow of a linearly viscous fluid between two rotating disks about two distinct vertical axes. Ashraf and Wehgal [15] solved numerically the problem of axisymmetric steady laminar incompressible flow of a micropolar fluid between two stationary infinite parallel porous disks with uniform injection through the surface of the disks. Abhijit Das [16] studied the steady flow of an incompressible viscous fluid between two infinite rotating coaxial disks and an exact analytical solution is obtained by using a Homotopy analysis method. Analysis study by using the Homotopy perturbation method for the two dimensional MHD flow and heat transfer of Casson fluid between porous disks is carried out by Devaki, et al. [17]. A lot of authors [18-20] done and introduced researches about the MHD flow between two disks.

So the main purpose of this paper is analyzing the MHD flow and the heat transfer of a steady incompressible viscous electrically conducting fluid between two porous discs. The case of uniform suction on both discs will be considered and an approximate analytical technique will be used for the solution of the governing equations of the problem. The discussion and illustration of the figures in terms of the nondimensional flow parameters will be shown and the effect of these parameters on the velocity fields and the temperature are studied.

2. Formulation of the problem

Consider the steady incompressible flow of an electrically conducting fluid of viscosity μ , density ρ and with electrical conductivity σ between two parallel porous discs. The lower stationary disc is located at plane $z = 0$ while the upper disc is placed at $z = d$ and rotating with a constant angular velocity Ω . The fluid is extracted (suction) with a uniform velocity w_1. A uniform magnetic field is applied at the discs of the strength B_0 that directed parallel to the z- axis, $\underline{B} = B_0 \hat{z}$. Let us consider that the distance d between the two discs is very small compared to the radius of the discs, so we can neglect the edge effects. T_1 is the temperature at the lower disc while T_2 are considered to be the temperature at the upper disk with $T_1 > T_2$, so due to the temperature gradient between the two discs, the heat transfer effects will be discussed . The problem geometry is shown in Fig (1).

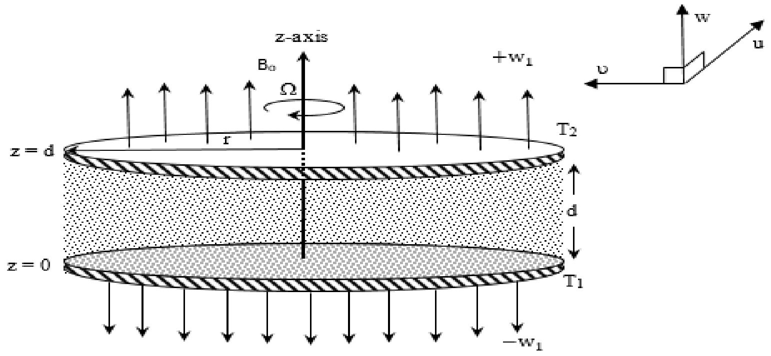

Fig (1) flow geometry and coordinate system

3. The basic governing equations

The governing equations of motion in case of the MHD steady viscous incompressible flow of the fluid in the free region $0 \le z \le d$ are:

a. The Navier-Stokes equation
$$\rho(\underline{v} \cdot \nabla)\underline{v} = -\nabla p + \mu\nabla^2\underline{v} + \underline{J} \times \underline{B}\,, \tag{1}$$
b. The continuity equation for an incompressible fluid
$$\nabla \cdot \underline{v} = 0\,, \tag{2}$$
c. The equation of the temperature field, by neglecting the viscous dissipation
$$(\underline{v} \cdot \nabla)T = \alpha\nabla^2 T. \tag{3}$$

Where the velocity vector of the fluid is $\underline{v}$, the gradient pressure is ∇p , ∇^2 is the Laplacian operator, $\underline{J}$ is the current density vector, $\underline{B}$ is the magnetic field vector, $\alpha = \frac{k}{\rho\, c_p}$ is the thermal diffusivity, k is thermal conductivity of fluid, c_p is the specific heat at constant pressure and $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity of the fluid.

The velocity vector $\underline{v}$ in the cylindrical polar coordinates which the suitable system for our problem may be written as following:
$$\underline{v} = u\,\hat{r} + \upsilon\,\hat{\theta} + w\,\hat{z}. \tag{4}$$

The three dynamical equations of motion in a component form by considering the flow is symmetrical around z-axis (any derivatives with respect to θ equal zero) are taken as:
$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{5a}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{\upsilon^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right] - \frac{\sigma B_0^2 u}{\rho}\,, \tag{5b}$$

$$u\frac{\partial \upsilon}{\partial r} + w\frac{\partial \upsilon}{\partial z} + \frac{u\upsilon}{r} = \nu\left[\frac{\partial^2 \upsilon}{\partial r^2} + \frac{1}{r}\frac{\partial \upsilon}{\partial r} + \frac{\partial^2 \upsilon}{\partial z^2} - \frac{\upsilon}{r^2}\right] - \frac{\sigma B_0^2 \upsilon}{\rho}, \tag{5c}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right], \tag{5d}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right). \tag{5e}$$

Where ∇^2 in the cylindrical polar coordinates (r, θ, z) is $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

The governing equations of the fluid flow have the corresponding boundary conditions:

At $z = 0, u = 0, \upsilon = 0, w = -w_1, T = T_1.$ (6a)

At $z = d, u = 0, \upsilon = r\,\Omega, w = w_1, T = T_2.$ (6b)

By using the following similarity transformation, that similar to Chandrasekhara et al. [6] , Kumar et al. [11] and Von Karman [1] to simplify the set of partial differential Equations (5a) to (5e) to be in the form of dimensionless ordinary differential equations,

$F'(\eta) = u\frac{2\,d}{w_1 r}$, $G(\eta) = \frac{\upsilon}{\Omega\, r}$, $F(\eta) = -\frac{w}{w_1}$, $\theta(\eta) = \frac{T-T_2}{T_1-T_2}$, (7a)

$p = -\rho r^2 \frac{{w_1}^2}{4d^2} + \frac{\mu w_1}{d} P(\eta)$. The new independent variable $\eta = \frac{z}{d}$ (7b)

The prime on $F'(\eta)$denotes differentiation with respect to η.

The velocity components in Eq (7a) satisfying the continuity Eq (5a). By using Eqs (7a) and (7b) into the governing Eqs (5b) to (5e), we get the following equations in the dimensionless form:

$F''''(\eta) = R[-F(\eta)F'''(\eta) - 2\lambda G(\eta)G'(\eta)] + M^2 F''(\eta),$ (8a)

$G''(\eta) = R[F'(\eta)G(\eta) - F(\eta)G'(\eta)] + M^2 G(\eta),$ (8b)

$P'(\eta) = -[R\, F(\eta)F'(\eta) + F''(\eta)]$ (8c)

$\theta''(\eta) = -\Pr R\, F(\eta)\, \theta'(\eta).$ (8d)

Where $M = \sqrt{\frac{\sigma {B_o}^2 d^2}{\nu\, \rho}}$ is the magnetic parameter (Hartmann number), $R = \frac{w_1\, d}{\nu}$ is the suction Reynolds number, $R_1 = \frac{\Omega\, d^2}{\nu}$ is the rotational Reynolds number, $\lambda = 2\frac{{R_1}^2}{R^2}$ is the ratio of Reynolds numbers and Pr = $\frac{\alpha}{\nu}$ is known as the Prandtl number. In the present problem we will discuss the suction case on both the two discs i.e. $R > 0$.

The pressure term will be estimated by the integration of Eq (8c) with respect to η:

$\Delta P = \frac{R}{2}[1 - F^2] - F'$ (8e)

Where $\Delta P = P(\eta) - P_0$ is the dimensionless pressure difference distribution and P_0 is the pressure at the stationary porous disc at $\eta = 0$.

The boundary conditions Eqs (6a) and (6b) must be reduced to the dimensionless form as:

At $\eta = 0, F' = 0, G = 0, F = 1, \theta = 1.$ (9a)

At $\eta = 1, F' = 0, G = 1, F = -1, \theta = 0.$ (9b)

4. Solution of the problem

As we mentioned before, the equations (8a), (8b) and (8d) are a set of nonlinear ordinary differential equations and haven't reached to an exact analytical solution until now for these Eqs, and all of the tries which achieved are either an approximate analytical solution or a numerical solution. We will seek an approximate solution to these equations that subjected to the boundary conditions Eqs (9a) and (9b) by using the perturbation method that depends on the following assumptions:

- **The Eqs (8a), (8b) and (8d) can be rewritten as:**

$\frac{F''''(\eta)}{M^2} = \frac{R}{M^2}[-F(\eta)F'''(\eta) - 2\lambda G(\eta)G'(\eta)] + F''(\eta)$, (10a)

$\frac{G''(\eta)}{M^2} = \frac{R}{M^2}[F'(\eta)G(\eta) - F(\eta)G'(\eta)] + G(\eta)$, (10b)

$\frac{\theta''(\eta)}{M^2} = -\frac{R}{M^2} \Pr F(\eta)\theta'(\eta)$. (10c)

- **For the case of uniform suction on the two discs, let $N = \frac{M^2}{R}$ and If assumed the solution for the case of $R << M^2$ $\therefore \frac{1}{N} << 1$, so Eqs (10a) to (10c) can be rewritten as:**

$\frac{F''''(\eta)}{M^2} = \frac{1}{N}[-F(\eta)F'''(\eta) - 2\lambda G(\eta)G'(\eta)] + F''(\eta),$ (11a)

$\frac{G''(\eta)}{M^2} = \frac{1}{N}[F'(\eta)G(\eta) - F(\eta)G'(\eta)] + G(\eta)$, (11b)

$\frac{\theta''(\eta)}{M^2} = -\frac{1}{N} \Pr F(\eta)\theta'(\eta).$ (11c)

The unknown functions$F(\eta)$, $G(\eta)$ and $\theta(\eta)$ can be expanded in successive powers of $\frac{1}{N}$ as following:

$$F(\eta) = F_0(\eta) + \frac{1}{N} F_1(\eta) + \frac{1}{N^2} F_2(\eta) + \ldots\ldots, \quad (12a)$$

$$G(\eta) = G_0(\eta) + \frac{1}{N} G_1(\eta) + \frac{1}{N^2} G_2(\eta) + \ldots\ldots, \quad (12b)$$

$$\theta(\eta) = \theta_0(\eta) + \frac{1}{N} \theta_1(\eta) + \frac{1}{N^2} \theta_2(\eta) + \ldots\ldots \quad (12c)$$

For simplicity of the solution, let's take only the first two terms in the previous expansion and the higher order terms in Eqs (12a) to (12c) are complicated algebraically and noted that the effects of the higher order terms are negligible in comparison with the zero and first order terms. So we will restricted in our analysis only to the zero and first order solutions.

Substituting Eqs (12a), (12b) and (12c) into Eqs (11a), (11b) and (11c) then collecting the coefficients of the same powers of $\frac{1}{N^0}, \frac{1}{N}$ we get:

The zero-order approximation:

$$F_0'''' - M^2 F_0'' = 0, \quad (13a)$$

$$G_0'' - M^2 G_0 = 0, \quad (13b)$$

$$\theta_0'' = 0. \quad (13c)$$

The first-order approximation:

$$F_1'''' - M^2 F_1'' = M^2[-F_0 F_0''' - 2\lambda G_0 G_0'], \quad (14a)$$

$$G_1'' - M^2 G_1 = M^2[F'_0 G_0 - F_0 G_0'], \quad (14b)$$

$$\theta_1'' = -M^2 Pr F_0 \theta_0'. \quad (14c)$$

The corresponding boundary conditions on the functions $F_n(\eta), G_n(\eta)$ and $\theta_n(\eta)$ are reduced to:

$$F_0(0) = 1, F_0(1) = -1, F'_0(0) = F'_0(1) = 0, \quad (15a)$$

$$F_1(0) = F_1(1) = F'_1(0) = F'_1(1) = 0, \quad (15b)$$

$$G_0(0) = 0, G_0(1) = 1, G_1(0) = G_1(1) = 0, \quad (15c)$$

$$\theta_0(0) = 1, \theta_0(1) = 0, \theta_1(0) = \theta_1(1) = 0. \quad (15d)$$

Zeroth order solutions: the zero-order solutions of the axial, radial and tangential velocity components and the temperature distribution which satisfying the boundary conditions Eqs (15a) to (15d) are:

$$F_0 = \frac{-1}{2} A_1[2(e^{M(1-\eta)} - e^{M\eta}) + M(e^M + 1)(2\eta - 1)]. \quad (16a)$$

$$F'_0 = A_1 M[(e^{M\eta} - e^{M(1-\eta)}) - (e^M + 1)]. \quad (16b)$$

$$G_0 = A_5(e^{M\eta} - e^{-M\eta}). \quad (16c)$$

$$\theta_0 = (1 - \eta). \quad (16d)$$

First order solutions: after some algebraic calculations, the first order solutions of the velocity and the temperature distribution Eqs (14a) to (14c) that satisfying the boundary conditions Eqs (15) are:

$$F_1 = (K_4\eta^2 + K_5\eta + D_1)e^{M\eta} + (K_2\eta^2 + K_3\eta + D_2)e^{-M\eta} + D_3\eta + D_4 + K_1 e^{-2M\eta} + K_6 e^{2M\eta}. \quad (17a)$$

$$F'_1 = D_3 - 2MK_1 e^{-2M\eta} - M(K_2\eta^2 + K_3\eta + D_2)e^{-M\eta} + (2K_2\eta + K_3)e^{-M\eta} + M(K_4\eta^2 + K_5\eta + D_1)e^{M\eta} + (2K_4\eta + K_5)e^{M\eta} + 2MK_6 e^{2M\eta}. \quad (17b)$$

$$G_1 = (N_6\eta^2 + N_7\eta + N_1)e^{M\eta} + (N_3\eta^2 + N_4\eta + N_2)e^{-M\eta} + N_5, \quad (17c)$$

$$\theta_1 = PrM^2\left[\frac{A_1 e^{M\eta}}{M^2} + \frac{A_2 e^{-M\eta}}{M^2} + \frac{A_3\eta^3}{6} + \frac{A_4\eta^2}{2} + t_1\eta + t_2\right]. \quad (17d)$$

Table (1) the values of all constants that satisfying the boundary conditions Eqs (15) to (15d) have been determined in the following table.

$A_1 = \frac{2}{(M+2)+e^M(M-2)}$	$A_2 = -e^M A_1$	$A_3 = -M(1+e^M) A_1$
$A_4 = \frac{-1}{2} A_3$	$A_5 = \frac{1}{(e^M - e^{-M})}$	$K_1 = \frac{-M A_1^2 B_1}{12}$
$K_2 = \frac{M^2 A_1^2 B_2 e^M}{2}$	$K_3 = \frac{-(M-5)}{M} K_2$	$K_4 = -K_2 e^{-M}$
$K_5 = \frac{M(5+M)A_1^2 B_2}{2}$	$K_6 = \frac{-M A_1^2 B_3}{12}$	$B_1 = -e^{2M} - \frac{2\lambda A_5^2}{M^2 A_1^2}$
$B_2 = \frac{-1}{2} M(e^M + 1)$	$B_3 = 1 + \frac{2\lambda A_5^2}{M^2 A_1^2}$	$N_1 = \frac{-N_5 e^{-M} + N_8}{(e^{-M} - e^M)}$
$N_2 = \frac{-N_8 + e^{\alpha} N_5}{(e^{-\alpha} - e^{\alpha})}$	$N_3 = \frac{-M^3 A_1 A_5 C_1}{4}$	$N_4 = \frac{-(M-3)}{M} N_3$
$N_5 = -M A_1 A_5 C_2$	$N_6 = -N_3$	$N_7 = \frac{-N_3(-3-M)}{M}$
$N_8 = (N_3 + N_4)e^{-M} + N_5 + (N_6 + N_7)e^M$	$C_1 = (e^M + 1)$	$C_2 = 2(e^M - 1)$

$$t_1 = -\left[\frac{A_1 e^M}{M^2} + \frac{A_2 e^{-M}}{M^2} + \frac{A_3}{6} + \frac{A_4}{2} + t_2\right] \qquad t_2 = -\left[\frac{A_1}{M^2} + \frac{A_2}{M^2}\right]$$

$$D_1 = \frac{A_1^2 M^2 E_1 + 2A_5^2 e^{-2M} E_2}{D_0} \qquad D_2 = \frac{A_1^2 M^2 e^M E_3 - 2A_5^2 e^{-M} E_4}{D_0}$$

$$D_3 = \frac{2M^3 A_1^2 E_5 - 2M A_5^2 e^{-2M} E_6}{D_0} \qquad D_4 = \frac{-A_1^2 M^3 E_7 + 2A_5^2 e^{-2M} E_8}{D_0}$$

$$D_0 = 12\, M(2 + e^M(-2+M) + M)$$

$$E_1 = (2 + 17M + 3M^2 + 15e^M M^2 + e^{2M}(-2 - 13M + 12M^2))$$

$$E_2 = \lambda(-1 - 2e^M M + 2e^{3M} M + 2e^{2M}(2+M) + e^{4M}(-3+2M))$$

$$E_3 = \left(-2 + 13M + 12M^2 + 15e^M M^2 + e^{2M}(2 - 17M + 3M^2)\right)$$

$$E_4 = \lambda(3 + e^{4M} + 2e^{2M}(-2+M) + 2M + 2e^M M - 2e^{3M} M)$$

$$E_5 = (16 + 3M + e^{3M}(-16+3M) + 3e^{2M}(-4+9M) + 3e^M(4+9M))$$

$$E_6 = \lambda(-1 + e^M)^3(1 + e^{2M})$$

$$E_7 = (16 + 3M + e^{3M}(-16+3M) + 3e^{2M}(-4+9M) + 3e^M(4+9M))$$

$$E_8 = \lambda(-1 + e^M)(-1 + e^{4M} - 4e^{3M}(-1+M) - 4e^{2M} M - 4e^M(1+M))$$

5. Results and discussion

It's clear that From the present problem solution, the velocity and the temperature distribution of the fluid flow are depended on the following flow parameters namely ,Hartmann number M, suction Reynolds number R, rotational Reynolds number R_1 and the Prandtl number Pr.

Figures 2-5 illustrate according to the approximation analytical results, the effect of the Hartmann number M on the axial, radial, tangential velocities and temperature distribution. By increasing the values of M, the axial velocity F(η) decreases near to the lower disc until the central plane then it increases near the upper disc as shown in Fig(2). With increasing M the radial velocity decreased at $\eta \le 0.24$ again through $\eta \ge 0.75$ then increased in the interval $0.24 < \eta < 0.75$ and the maximum value of The radial velocity F′(η) shifts towards the rotating disc and has a minimum value at the midplane, also noted that F'(η) nearly symmetrical around $\eta = 0.5$,see Fig(3).The effect of increasing M is to decrease the tangential velocity G(η) as shown in Fig(4). From Fig (5) it can be noticed that the temperature of the fluid decreases over with increasing η for a given M. The effect of increasing M on the temperature is to slightly increase it for $\eta < 0.5$ and the stability occurs at the midplane of the two discs but near to the upper disk it decreases at $\eta > 0.5$.

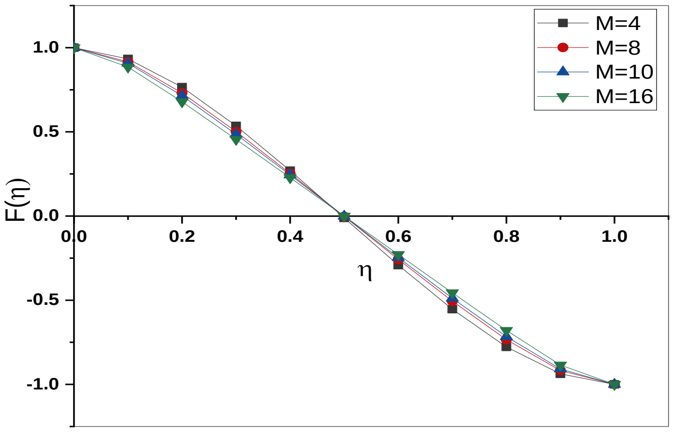

Fig. (2) Effect of different values of M on the axial velocity F(η) at R= 0.5 and R_1 = 2.

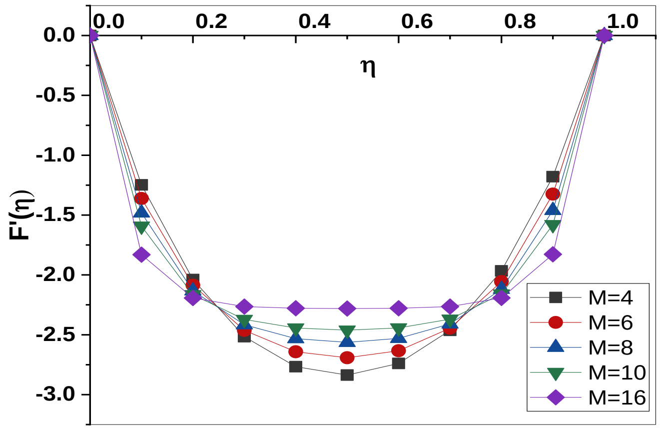

Fig. (3) Effect of different values of M on the radial velocity F′(η) at R= 0.5 and R_1 =2 .

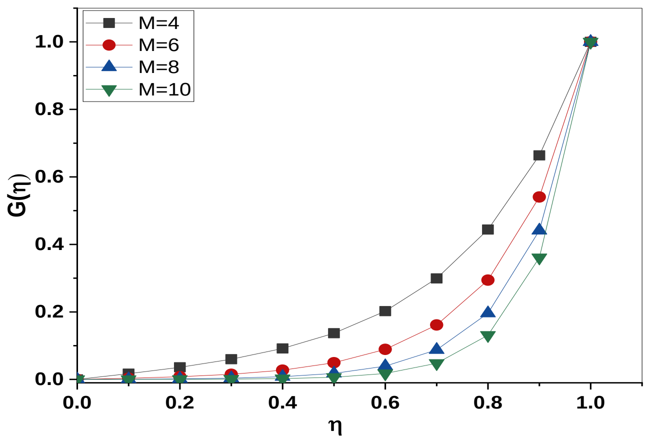

Fig. (4) Effect of different values of M on the tangential velocity G (η) at R= 0.5.

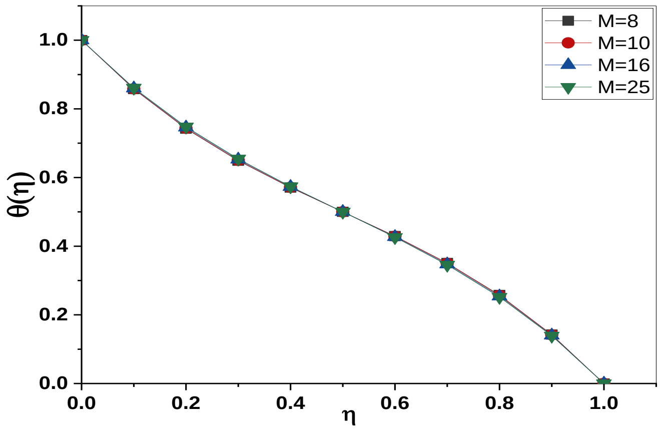

Fig. (5) Effect of different values of M on the temperature θ(η) at R=1 and Pr =3.

Figures 6-9 show the effect of the suction Reynolds number R on the axial, radial and tangential velocities and temperature distribution. With increasing values of the suction Reynolds number R noticed that, the axial velocity F(η) will be increased as shown in Fig(6). By increasing suction velocity through the discs, the radial velocity F′(η) increased near to the lower disc until $\eta = 0.6$ then decreased and falls to zero at the upper disc as shown in Fig (7). With the increase in R values, the tangential velocity G(η) increases until the value of $\eta < 0.7$ then decreases in the area nearer to the upper disk as shown in Fig(8). Fig (9) illustrates that when the values of R increased, a decrease in the temperature near to the lower disc at $\eta < 0.5$ is observed and above $\eta > 0.5$ an increase in the temperature distribution reached.

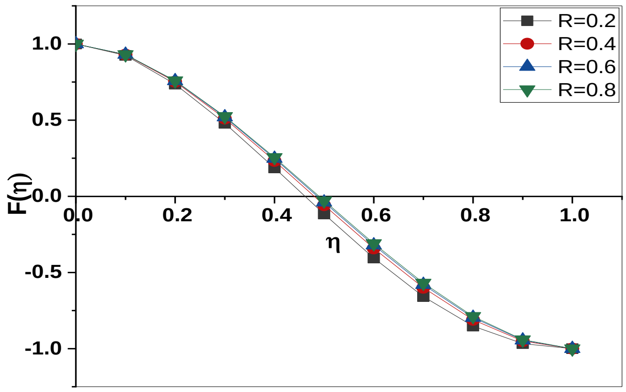

Fig. (6) Effect of different values of R on the axial velocity F(η) at M= 4 and R_1= 4.

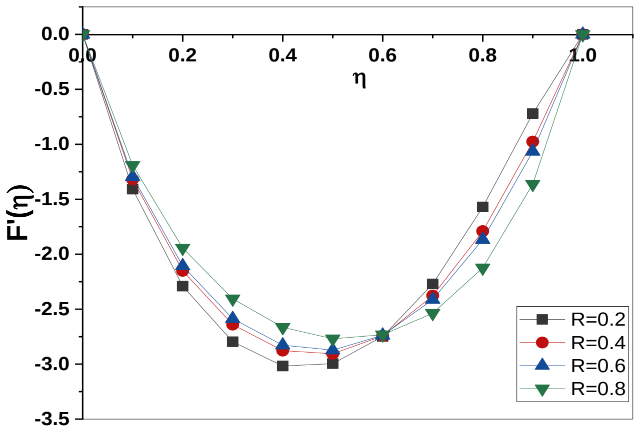

Fig. (7) Effect of different values of R on the radial velocity F′(η) at M= 4 and R_1= 4.

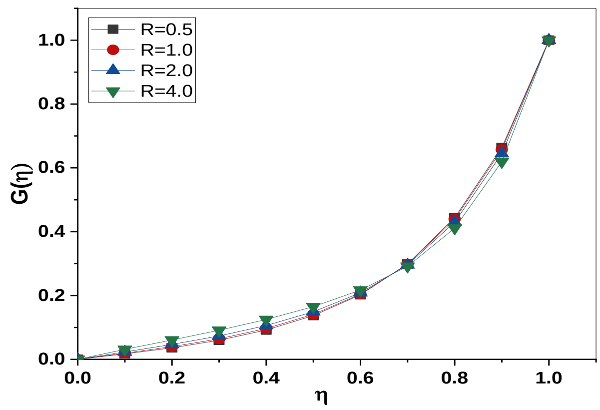

Fig. (8) Effect of different values of R on the tangential velocity $G(\eta)$ at M= 4.

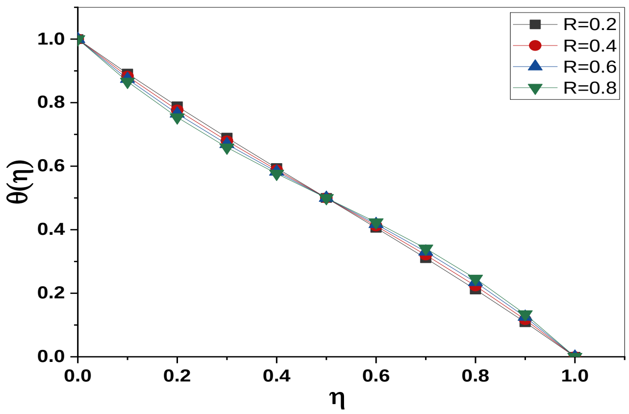

Fig. (9) Effect of different values of R on the temperature $\theta(\eta)$ at M= 8 and Pr =3.

Figures 10-11 indicate the effect of rotational Reynolds number R_1 on the axial and radial velocity. The axial velocity $F(\eta)$ decreased with increasing R_1 as shown in Fig(10) . The radial velocity $F'(\eta)$ decreased near to the lower disc until $\eta = 0.6$ then increased and the maximum value of the velocity shifts towards the upper disc by increasing the value of R_1 as shown in Fig. (11). According to our present approximating solutions noticed that tangential velocity and temperature distribution not dependent on R_1.

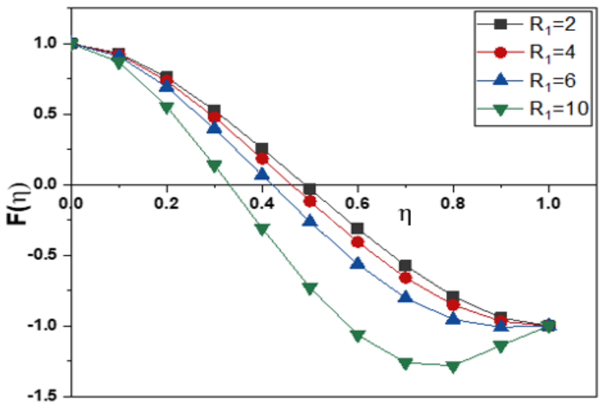

Fig. (10) Effect of different values of R_1 on the axial velocity $F(\eta)$ at M = 4 and R= 0.2.

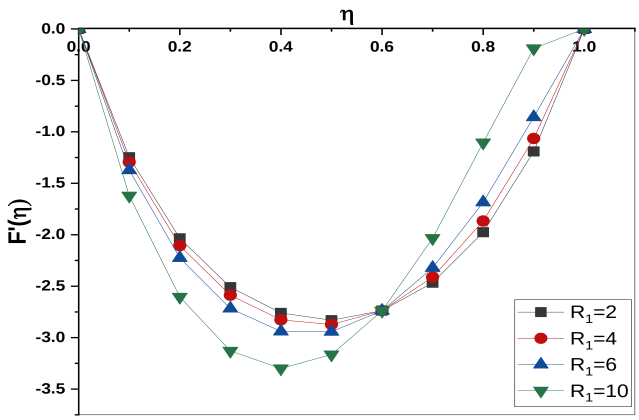

Fig. (11) Effect of different values of R_1 on the radial velocity $F'(\eta)$ at M= 4 and R= 0.6.

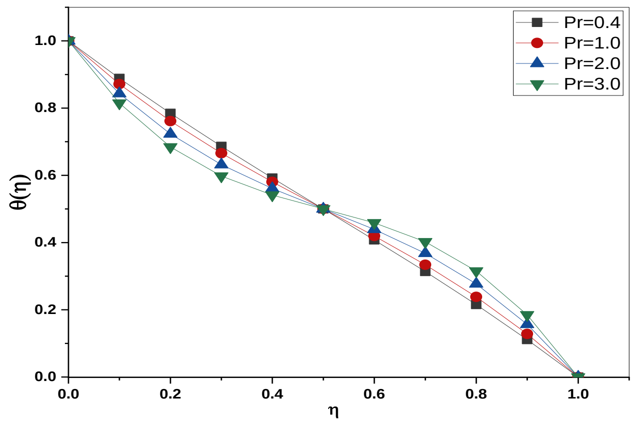

Fig. (12) Effect of different values of Pr on the temperature $\theta(\eta)$ at M= 8 and R =2.

Figure (12) illustrates the effect of Prandtl number Pr on the temperature distribution. With increasing the values of Pr will cause a decrease in the temperature near to the lower disc and again the stability occurs at the midplane at $\eta = 0.5$ whereas an increase in the temperature profiles near to the upper disc.

6. Conclusion

In The present work, the effects of governing parameters on the steady MHD flow and heat transfer process of an incompressible viscous and electrically conducting fluid between two discs have been investigated. Approximate analytical solution of the transformed governing equations has been obtained. From the results of the analytical computations, the following conclusions have been extracted as the following:

A. By increasing the magnetic parameter M values, the axial velocity falls near to lower disc and increased near the upper disc and the maximum value of the radial velocity shifts towards the rotating disc and has a minimum value at the midplane of the discs. The effect of increasing M is to decrease the tangential velocity and the temperature slightly increased near to the lower disc but near the upper disc it slightly decreased.

B. As the values of suction Reynolds number R increased, the axial velocity is increased, while the radial velocity increased near the lower disc then decreased at $\eta > 0.6$, the tangential velocity increased until the value of $\eta < 0.7$ but decreased nearer to the upper disk area and the temperature distribution has been decreased near to the lower disc while an increase in the temperature above $\eta > 0.5$ has been noticed.

C. Increasing the values of the Prandtl number caused a decrease in the temperature profiles in a region towards lower disk at $\eta < 0.5$ but results in an increase in temperature profiles in a region after the central plane towards the upper disk. Noticed that the effect of Pr formed as an effective insulating layer at the midplane of the disks (flattening of the temperature profile).

References

[1] T. V. Karman, Uber laminare und turbulente Reibung, Z. Angew. Math. Mech. Vol. 1, PP. 233-252, 1921.

[2] G.K. Batchelor, Note on a class of solutions of the Navier-Stokes equations representing steady rotationally-symmetric flow, The

quarterly journal of mechanics and applied mathematics. Vol. 4, PP. 29-41, 1951.
[3] A. R. Elcrat, On the radial flow of a viscous fluid between porous disks, Archive for Rational Mechanics and Analysis. Vol. 61, PP. 91–96, 1976.
[4] H. Rasmussen, Steady viscous flow between two porous disks, Zeitschrift für angewandte Mathematik und Physik ZAMP . Vol.21, PP. 187–195, 1970.
[5] Y. N. Guar, R. C. Chaudhary, Heat transfer for laminar flow through parallel porous disks, DEF. SCI.J. Vol. 30, PP. 153–158, 1980.
[6] N. Rudraiah, B. C. Chandrasekhara, Three dimensional magnetohydrodynamic flow between two porous disks, Applied scientific research. Vol. 25, PP. 179-192, 1972.
[7] K. Stewartson, On the flow between two rotating coaxial disks, In Mathematical Proceedings of the Cambridge Philosophical Society .Vol. 49, PP. 333–341, 1953.
[8] C.Y. Wang, L. T.Watson, Viscous flow between rotating discs with injection on the porous disc, Zeitschrift für angewandte Mathematik und Physik ZAMP. Vol. 30, PP. 773-787, 1979.
[9] A. C. Srivastava, S.K. Sharma, Effect of transverse magnetic field on flow between two infinite discs-one rotating and other at rest, Bull. Acad. Pol. Sci., Ser. Sci. Tech.Vol. 9, PP. 639, 1961.
[10] C. J. Stephenson, Magnetohydrodynamic flow between rotating coaxial disks, Journal of Fluid Mechanics .Vol. 38, PP. 335-352,1969.
[11] K.S. Kumar, W.I. Thacker, L.T. Watson, Magnetohydrodynamic flow between a solid rotating disk and a porous stationary disk, Appl. Math. Modelling. Vol. 13, PP. 494-500, 1989.
[12] R.S. Agarwal, R. Bhargava, A numerical study of Magnetohydrodynamic flow between a rotating and a stationary porous coaxial discs, Proc Indian Acad. Sci. Vol. 88, PP. 399-407, 1979.
[13] F.N. Ibrahim, Unsteady flow between two rotating disks with heat transfer, Journal of Physics D: Applied Physics. Vol. 24, PP. 1293-1299, 1991.
[14] H.V. Ersoy, An approximate solution for flow between two disks rotating about distinct axes at different speeds, Mathematical Problems in Engineering.Vol. 2007, PP. 1-16, 2007.
[15] M. Ashraf, A.R. Wehgal, MHD flow and heat transfer of micropolar fluid between two porous disks, Applied mathematics and mechanics.Vol. 33, PP. 51-64, 2012.
[16] D. Abhijit, Analytical solution to the flow between two coaxial rotating disks using HAM, Procedia Engineering.Vol. 127, PP. 377-382, 2015.
[17] B. Devaki, P.P.Nityanand , V. S.S.Kumar, Analysis of MHD Flow and Heat Transfer of Casson fluid Flow Between Porous Disks, Journal of Advanced Research in Fluid Mechanics and Thermal Science. Vol. 83, PP.46-60, 2021.
[18] G. N. Purohlt, R.D. Sharma, MHD-heat transfer due to flow between two rotating porous discs, INDIAN JOURNAL OF PURE & APPLIED MATHEMATICS. Vol. 16, PP. 1377-1389, 1985.
[19] A. Fatma, Y. Hayriye, A .Nilay, Approximate solution for Magnetohydrodynamics fluid flow between two circular discs, International Journal of Mechanical Engineering. Vol. 3, PP.1-8, 2018.
[20] T. Hayat, S. Qayyum, M. Imtiaz, A. Alsaedi, MHD flow and heat transfer between coaxial rotating stretchable disks in a thermally stratified medium, PloS one. Vol.11, PP. 1-23, 2016.